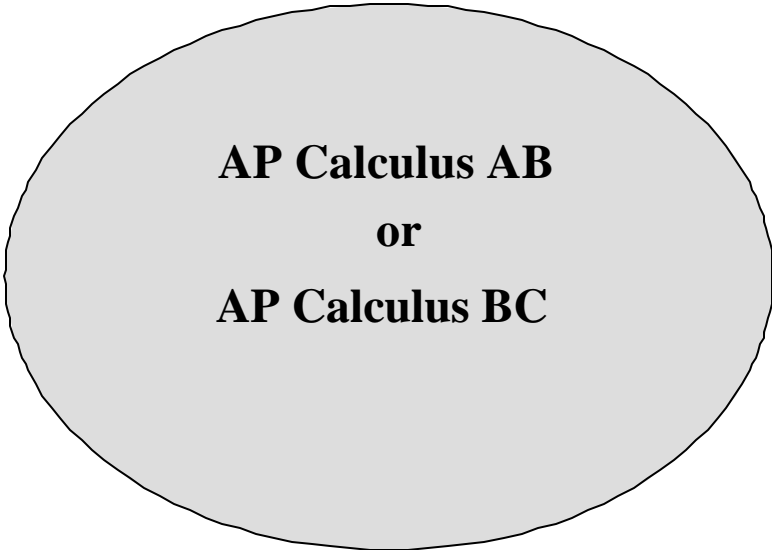


Name: _____

Period: ____ Date: _____

Lake Braddock Secondary School
Mathematics Department
Summer Packet



**AP Calculus AB
or
AP Calculus BC**

For students entering AP Calculus AB or AP Calculus BC

Show all work!!! Answers without justification are unacceptable

Functions

A **function** is a rule that assigns to each member in its **domain** a unique member in the **range**.

Example: $f(x) = x^2(x-3)(x+2)$

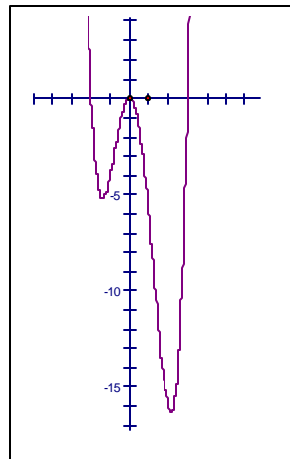
For domain value of -1 , the range value would be -4 ,
written $f(-1) = -4$

The **zeros (x-intercepts)** of $f(x)$ are $-2, 0, 3$ (set $y = 0$)

The **y-intercept** (set $x = 0$) of $f(x)$ is 0

The **domain** of $f(x)$ is all real numbers

The **range** of $f(x)$ is all real numbers



Complete the following exercises:

1. Given: $f(x) = -|x+3| - 2$

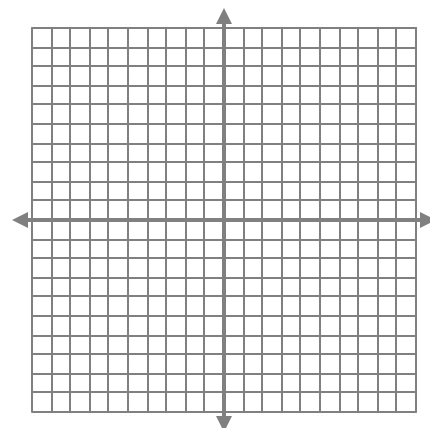
a. Graph $f(x)$.

b. Domain: _____

c. Range: _____

d. $f(3) =$ _____

e. If $f(x) = -3$, then $x =$ _____



2. Given the piecewise function: $g(x) = \begin{cases} x/2, & \text{if } x \geq 4 \\ \sqrt{x}, & \text{if } 0 < x < 4 \\ x^2, & \text{if } x < 0 \end{cases}$

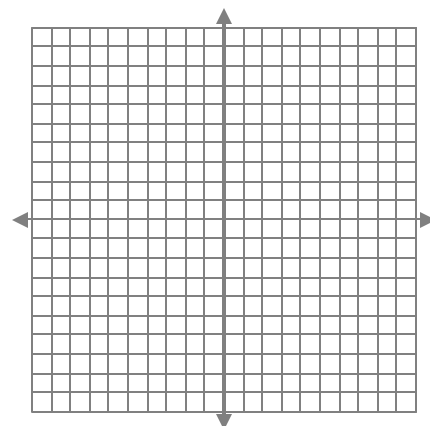
a. Graph $g(x)$.

b. $g(-3) =$ _____

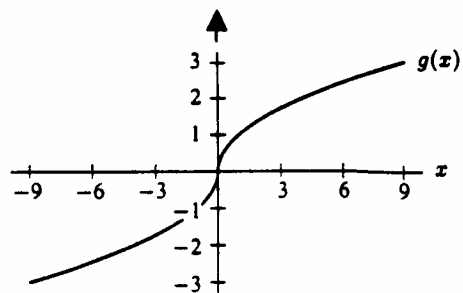
c. $g(1) =$ _____

d. $g(0) =$ _____

e. Is $g(x)$ a continuous function? Explain.



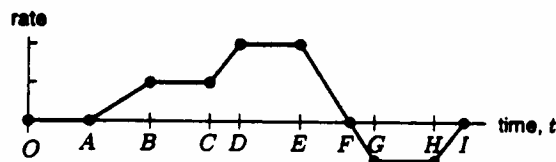
3. Given the graph of $g(x)$ on the right



a. Estimate $\frac{g(6) - g(0)}{6 - 0}$

b. The ratio in part (a) is the slope of a line segment joining two points on the graph. Sketch this line segment.

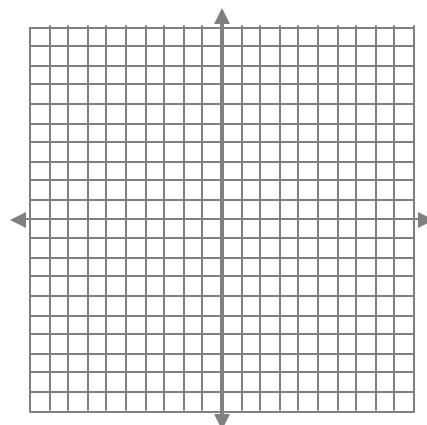
4. The rate at which water is entering a tank ($t > 0$) is represented by the given graph. A negative rate means that water is leaving the tank. State the interval(s) on which each of the following holds true:



- The volume of water is constant _____
- The volume of water is decreasing _____
- The volume of water is increasing _____
- The volume of water is increasing fastest _____

5. $Q(x) = \frac{3x}{x+1}$

- Where is this function discontinuous? _____
- State the equation of the vertical asymptote _____
- State the equation of the horizontal asymptote _____
- Sketch
- Write the equation of the inverse of $Q(x)$ _____

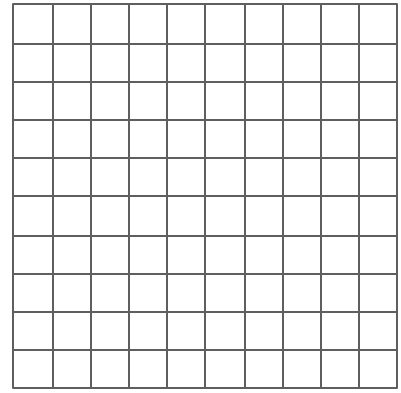


TRIG TOPICS

- The natural real number domain of the sine function is _____; its range is _____.
- The period of the cosine function is _____; the period of the sine function is _____; the period of the tangent function is _____.
- Of the six trigonometric functions, _____ are odd functions and _____ are even functions.
- If $(-4, 3)$ lies on the terminal side of an angle θ whose vertex is at the origin and initial side is along the positive x -axis, then $\cos \theta =$ _____ (fraction) .
- Convert the following degree measures to radians (leave π in your answer).
 - $240^\circ =$ _____
 - $-60^\circ =$ _____
 - $-135^\circ =$ _____
 - $540^\circ =$ _____
 - $600^\circ =$ _____
 - $720^\circ =$ _____
 - $18^\circ =$ _____
 - $22.5^\circ =$ _____
- Convert the following radian measures to degrees.
 - $\frac{7p}{6} =$ _____
 - $\frac{-p}{3} =$ _____
 - $8\pi =$ _____
 - $\frac{5p}{4} =$ _____
 - $\frac{3p}{2} =$ _____
 - $\frac{-11p}{12} =$ _____
 - $\frac{p}{18} =$ _____
 - $\frac{7p}{4} =$ _____
- Evaluate without use of a calculator.
 - $\tan\left(\frac{p}{6}\right) =$ _____
 - $\cos\left(\frac{-p}{3}\right) =$ _____
 - $\sin(p) =$ _____
 - $\csc\left(\frac{p}{2}\right) =$ _____
- Verify that the following are identities:
 - $(1 + \sin z)(1 - \sin z) = \frac{1}{\sec^2 z}$
 - $\frac{\sin u}{\csc u} + \frac{\cos u}{\sec u} = 1$
 - $\sec t - \sin t \tan t = \cos t$
- Find identities analogous to the addition identities for each expression.
 - $\sin(x - y)$
 - $\cos(x - y)$
 - $\tan(x - y)$

10. Draw the graphs of each of the following functions on $[-p, 2p]$ using the same axis.

(a) $f(x) = \sin x$ (b) $f(x) = 2\cos x$ (c) $f(x) = \tan \frac{x}{2}$



11. Find the exact values without use of a calculator.

(a) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$ (b) $\cos^{-1}\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$ (c) $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = \underline{\hspace{2cm}}$
 (d) $\arctan(-\sqrt{3}) = \underline{\hspace{2cm}}$ (e) $\sec^{-1}(-2) = \underline{\hspace{2cm}}$ (f) $\sin(\cos^{-1}(0.6)) = \underline{\hspace{2cm}}$

EIGHT FUNDAMENTAL IDENTITIES

Reciprocal Identities

1. $\csc q = \frac{1}{\sin q}, \sin q \neq 0$

4. $\sec q = \frac{1}{\cos q}, \cos q \neq 0$

7. $\cot q = \frac{1}{\tan q}, \tan q \neq 0$

Ratio Identities

2. $\tan q = \frac{\sin q}{\cos q}, \cos q \neq 0$

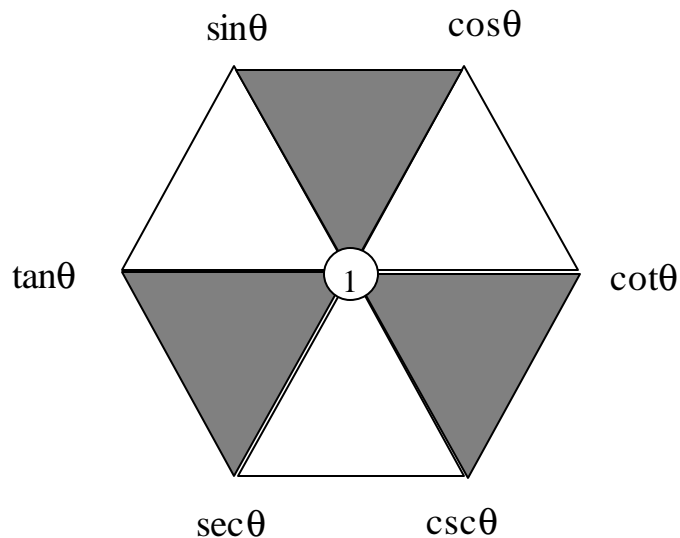
5. $\cot q = \frac{\cos q}{\sin q}, \sin q \neq 0$

Pythagorean Identities

3. $\sin^2 q + \cos^2 q = 1$

6. $1 + \tan^2 q = \sec^2 q$

8. $1 + \cot^2 q = \csc^2 q$



DERIVATIVES

The derivative is the limit of the slopes of a lot of secant lines. The limit of the slopes of the secant lines is the slope of the tangent line to the curve at a single point. Therefore, by transitivity, the “derivative of a function at a point is the slope of the tangent line at that single point. The notation $f'(x)$ is used to denote derivative. The derivative is a function just like $f(x)$, and is composed of ordered pairs $(x, f'(x))$. The formal definition of the derivative is a “limit” and is written:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

The Power Theorem is a method used for taking the derivative of a variable power.

$$\text{If } f(x) = x^n, \text{ then } f'(x) = nx^{n-1}$$

EXAMPLES: If $f(x) = x^4$, then $f'(x) = 4x^3$

$$\text{If } f(x) = x^{3/2}, \text{ then } f'(x) = 3/2 x^{1/2}$$

$$\text{If } f(x) = x^{0.9}, \text{ then } f'(x) = 0.9x^{-0.1}$$

PROBLEMS:

A. Find the derivatives of the following functions using the definition of derivative.

1) $f(x) = 2x$

2) $f(x) = x^2$

3) $f(x) = \frac{1}{x}$

B. Find the derivatives of the following functions using the power theorem.

1) $f(x) = x^3$

2) $f(x) = \sqrt{x}$

3) $f(x) = 6x^3$

4) $f(x) = x^{2/3}$

5) $f(x) = x^{-4}$

6) $f(x) = \frac{1}{x^3}$

Logarithms

Definition of Logarithm: $y = \log_b x \Leftrightarrow b^y = x$

Log of a product: $\log_b(xy) = \log_b x + \log_b y$

Log of a quotient: $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$

Log of a power: $\log_b x^n = n \log_b x$

Change - of - Base: $\log_a x = \frac{\log_b x}{\log_b a}$

Problems: Find x . Show work.

4. $\log_2 x = 3$

5. $\log_{\frac{1}{2}} x = 4$

6. $\log_3 81 = x$

7. $\log_3(-9) = x$

8. $\log_x 16 = -4$

1. $\log_x\left(\frac{1}{25}\right) = \frac{1}{2}$

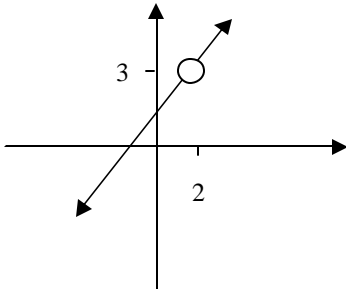
2. $2^x = 3$

3. $2.43 \cdot 10^x = 1.84$

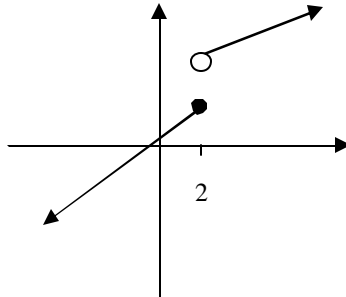
9. $\ln(x+5) = \ln(x-1) - \ln(x+1)$

Limits

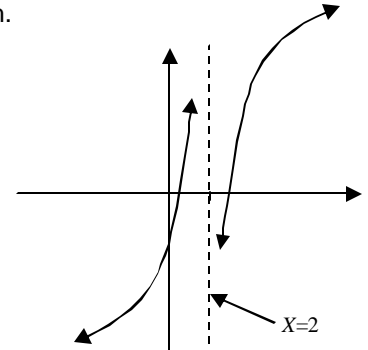
The limit of a function is the y-value that you are getting close to as x gets close to some number in the domain. In the "limit process" you never get to the limit, except for the limit of a constant function. We write $\lim_{x \rightarrow a} f(x)$, which is read "the limit of $f(x)$ as x approaches a domain value of a ." The limit must be the same as x approaches " a " from both the left and the right. To find the limit, substitute in values very close to " a " on both left and right and see if the y -value is approaching a single value. The limit does exist at a hole in a graph, but does not exist at a vertical asymptote or a jump in the graph.



Limit exists at 2



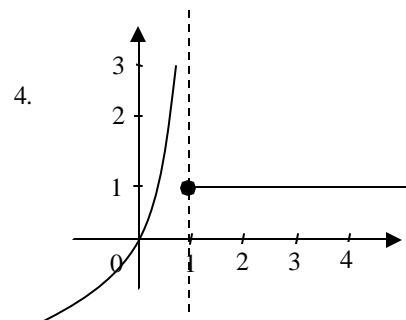
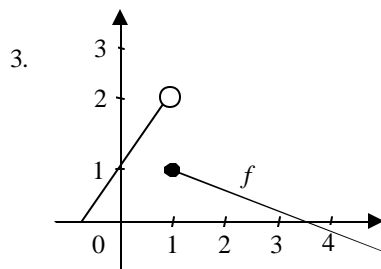
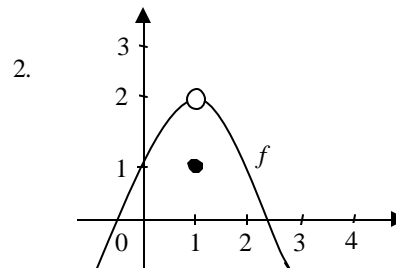
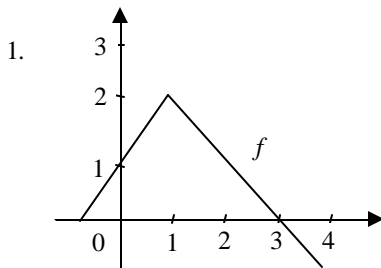
No limit at 2



No limit at 2

Problems:

The graphs of some functions are pictured below. Do you think that $\lim_{x \rightarrow 1} f(x)$ exists? If you think the limit does exist, state its value.



State the value of each of the following:

5. $\lim_{x \rightarrow 3} x = \underline{\hspace{2cm}}$

6. $\lim_{x \rightarrow -2} x = \underline{\hspace{2cm}}$

7. $\lim_{x \rightarrow 1} 2x = \underline{\hspace{2cm}}$

8. $\lim_{x \rightarrow -1} (x + 1) = \underline{\hspace{2cm}}$

9. $\lim_{x \rightarrow 2} x^2 = \underline{\hspace{2cm}}$

10. $\lim_{x \rightarrow -2} |x| = \underline{\hspace{2cm}}$