

Precalculus Summer Packet

These problems are to be completed to the best of your ability by the first day of school. You will be given the opportunity to ask questions about problems you found difficult during the first week of school. You will be tested on this information at the beginning of the second week of school. This packet will be graded as your first week's worth of homework. DO NOT WAIT until the night before to do this!! YOU'LL BE SORRY!

These problems should be familiar to you. They are review questions from Algebra I and Algebra II. Below is a list of topics covered by these problems and formulas you may need.

Linear Equations: $y = mx + b$, $y - y_1 = m(x - x_1)$

Intersection of two lines: (graphing, substitution, elimination)

Distance between two points: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of a line segment: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Slopes of Lines: $\frac{y_2 - y_1}{x_2 - x_1}$, parallel lines (have the same slope), perpendicular lines (have negative reciprocal slopes)

Linear functions: $f(x) = mx + b$

Complex numbers: $a + bi$, the number $i = \sqrt{-1}$, complex conjugate $a - bi$

Solving quadratic equations: factoring, quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, completing the square, the discriminant

Quadratic functions: parabolas, vertex, axis of symmetry, $y = ax^2 + bx + c$

For problems 1 and 2, find the length of \overline{CD} and the midpoint of \overline{CD} .

1. $C = (-8, -3)$, $D = (7, 5)$ 2. $C = (\frac{1}{2}, \frac{9}{2})$, $D = (2, \frac{3}{2})$

For problems 3 and 4, solve the given system of equations.

3. $\begin{cases} 2x + 3y = 15 \\ 4x - 9y = 3 \end{cases}$ 4. $\begin{cases} -2x - 6y = 18 \\ x - 3y = 6 \end{cases}$

5. Plot the points $A = (2, 0)$, $B = (4, -6)$, $C = (9, 1)$, and $D = (7, 7)$. Show that \overline{AB} and \overline{CD} bisect each other. What kind of quadrilateral is $ABCD$?

For problems 6 and 7, find the slope of the line joining the points whose coordinates are given.

6. $(-3, 8)$ and $(-3, -2)$ 7. $(-\frac{1}{3}, 2)$ and $(\frac{1}{4}, -\frac{1}{3})$

For problems 8 - 10, find the slope and the y-intercept of the line whose equation is given.

8. $y = 5$ 9. $y = 4 - 5x$ 10. $3x + 9y = 7$

11. Tell which of the given equations have parallel line graphs and which have perpendicular line graphs.

a. $3y = 5x - 5$ b. $y = -\frac{3}{5}x + 4$ c. $10y = -6x - 7$

12. Show that the line through $(2, -3)$ and $(7, 2)$ is perpendicular to the line through $(-3, 7)$ and $(2, 2)$.

13. Find the value of k if the line joining $(4, k)$ and $(6, 8)$ and the line joining $(-1, 4)$ and $(0, 8)$ are...

a. parallel b. perpendicular

For problems 14 - 17, write an equation of the line described.

14. The line with y-intercept -6 and parallel to the line $5x + 4y = 1$.

15. The line through $(-1, 4)$ and $(5, 8)$.

16. The horizontal line through $(5, -7)$.

17. The vertical line through $(5, -7)$.

18. a. If $f(x) = \frac{3}{4}x - \frac{1}{2}$, find $f(2)$ and $f(-2)$. b. Find the zero of $f(x)$.

19. Let $h(t) = \frac{9-4t}{2}$. Decide whether $h(4.5) - h(3.5) = h(1)$.

For problems 20 - 28, simplify each expression.

20. $\sqrt{-4} + \sqrt{-16} + \sqrt{-1}$

21. $\sqrt{-1} \cdot \sqrt{-9}$

22. $\frac{\sqrt{-12}}{\sqrt{-3}}$

23. $(4-3i) + (-6+8i)$

24. $4(3+5i) - 2(2-6i)$

25. $(6-i)(6+i)$

26. $(5+i\sqrt{5})(5-i\sqrt{5})$

27. $(8+3i)(2-5i)$

28. $(4-5i)^2$

For problems 29 - 34, write each expression in the form $a+bi$.

29. $\frac{1}{2+5i}$

30. $\frac{5+i}{5-i}$

31. $\frac{5}{i}$

32. $i+i^2+i^3+i^4+i^5$

33. i^{-3}

34. i^{-35}

For problems 35 and 36, solve the equation by factoring.

35. $3x^2 - 4x - 7 = 0$

36. $(2x-3)(x+4) = 0$

For problems 37 - 39, solve the equation by completing the square. Give both real and imaginary roots.

37. $x^2 - 10x = 1575$

38. $2z^2 - 16z - 1768 = 0$

39. $x^2 + 6x + 10 = 0$

For problems 40 - 42, solve the equation using the quadratic formula. Give your answers in simplest radical form. Give both real and imaginary roots.

40. $5x^2 + 2x - 1 = 0$

41. $3t^2 = 12t - 15$

42. $\frac{4}{v} = \frac{v-6}{v-4}$

For problems 43 - 45, sketch each parabola. Label the vertex, the axis of symmetry, and the x- and y-intercepts.

43. $y = -x^2 + 4x - 4$

44. $y = x^2 - 6x + 10$

45. $y = 2(x+3)^2 + 6$

The problems that follow are a preview of your first test. You probably want to try these!

1. Let $A = (-2, -6)$ and $B = (-4, 2)$. Find the length of \overline{AB} and the coordinates of the midpoint of \overline{AB} .
2. Find the value of a if the point $(4, -2)$ lies on the line $2x + ay = 14$.
3. Solve the equations $2x + 3y = 2$ and $6x - y = -4$ simultaneously. Then sketch the graphs of the lines and label the intersection point.
4. Find the slope and the y-intercept of the line $4x - 2y = 7$.
5. Tell which of the following equations have parallel line graphs and which have perpendicular line graphs.
 - a. $2x + 3y = 1$
 - b. $y = \frac{3}{2}x + 3$
 - c. $6x - 4y - 10 = 0$
6. Write an equation of the line through $(6, -2)$ and $(-3, 1)$.
7. Write an equation of the line through $(5, 5)$ and parallel to the line $4x + 3y = -2$.
8. Write an equation of the line with x-intercept -3 and y-intercept -5 .
9. Write an equation of the vertical line through $(4, -2)$.
10.
 - a. Describe, in a few sentences, the steps involved in finding an equation of the perpendicular bisector of a line segment.
 - b. Write an equation of the perpendicular bisector of the segment joining the points $(7, 0)$ and $(1, 8)$.

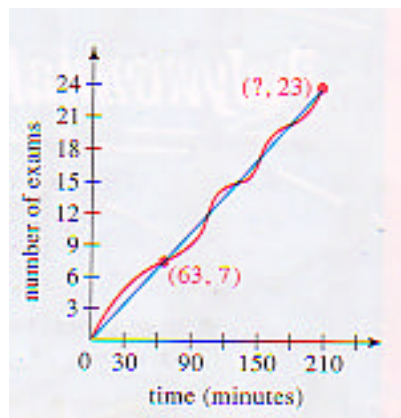
For problems 11 - 16, express each complex number in the form $a + bi$.

11. $\sqrt{-50} - \sqrt{-8}$
12. $(2 + 3i)^2$
13. $\frac{1}{2 + 3i}$
14. $\frac{\sqrt{3} + i}{\sqrt{3} - i}$
15. $4(3 + 2i) - 5(1 - i)$
16. i^{17}

17. A teacher must grade 23 exam papers. Working for 63 minutes, she is able to grade exactly seven exams. The function that models the number of exams graded is shown below in red. Because some exams take longer to grade than others, the teacher does not work at a constant rate. Nevertheless, her rate is close enough to being constant that a linear function (whose graph is the straight line shown in blue) can be used effectively to model her progress.

a. Give an equation of the linear function that models the number of exams graded as a function of time.

b. Use the equation to find approximately how long it will take to grade all the exams.



18. Solve each equation for x .

a. $7x^2 - 2 = 5x$

b. $x^2 - 4x = 9$

c. $x^2 + 2x + 2 = 0$

19. Sketch the graph of each parabola. Label the vertex, the axis of symmetry, and the intercepts.

a. $y = 8 - 2(x - 1)^2$

b. $y = x^2 - 6x + 5$

20. Sketch the graphs of the line $2x - y = -2$ and the parabola $y = -x^2 + 4x + 1$. Find the coordinates of any point(s) of intersection.