7.1 Rigid Motion in a Plane

VOCABULARY

Image An image is a new figure that results from the transformation of a figure in a plane.

Preimage A preimage is the original figure in the transformation of a figure in a plane.

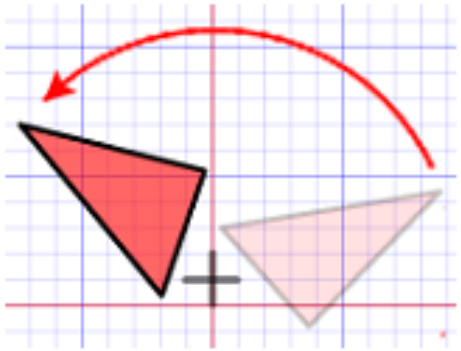
Transformation A transformation is the operation that maps, or moves, a preimage onto an image.

Isometry An isometry is a transformation that preserves lengths. Isometries are also called rigid transformations.

Transformations

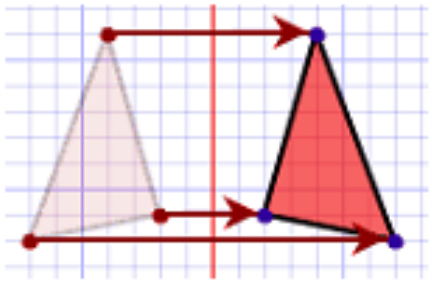
The three main Transformations are:

Rotation



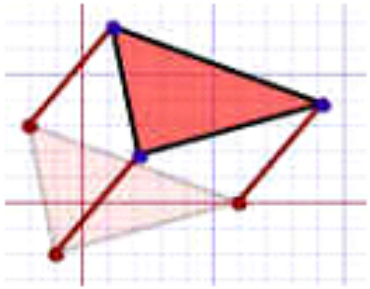
Turn!

Reflection



Flip!

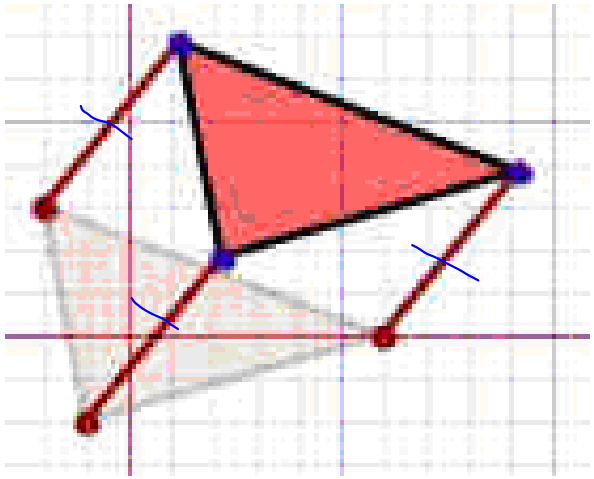
Translation



Slide!

Translation

... without rotating, resizing or anything else, just moving.



Every point of the shape must move:

- the **same distance**
- in the**same direction.**

After any of those transformations (turn, flip or slide), the shape still has **the same size**, **area**, **angles** and **line lengths.**

This is called an *isometry*.

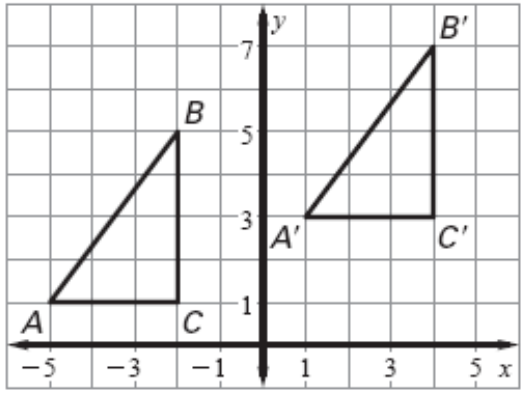
Example 1 *Naming Transformations*

Use the graph of the transformation at the right.

a. Name and describe the transformation.

b. Name the coordinates of the vertices of the image.

c. Is $\triangle ABC$ congruent to its image?



When you name an image, take the corresponding point of the preimage and add a prime symbol. For instance, if the preimage is A, the image is A', read as "A prime."

Solution

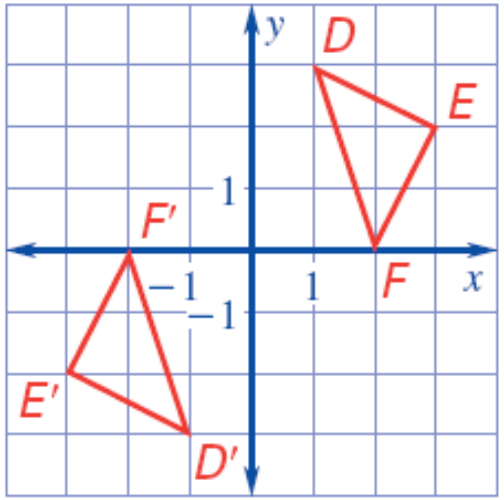
a. The transformation is a translation. You can imagine that the image was obtained by sliding $\triangle ABC$ up and to the right.

b. The coordinates of the vertices of the image, $\triangle A'B'C'$, are $A'(1, 3)$, $B'(4, 7)$, and $C'(4, 3)$.

c. Yes, $\triangle ABC$ is congruent to its image $\triangle A'B'C'$. One way to show this would be to use the Distance Formula to find the lengths of the sides of both triangles. Then use the SSS $\cong$ _____.

Another example:

Use the graph below.



a. Name and describe the transformation. rotation

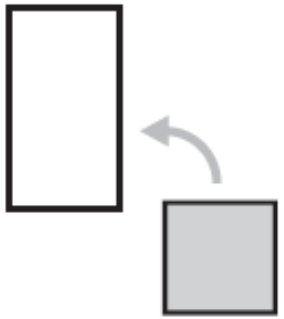
b. Name the coordinates of the vertices of the image. D'(-1,-3)
E'(-3,-2)
F'(-2,0)

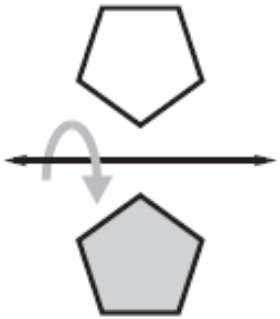
c. Is $\triangle DEF$ congruent to its image?

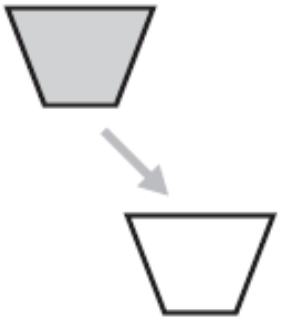
yes

Identifying Isometries

Does the transformation appear to be an isometry?

a. 

b. 

c. 

Solution

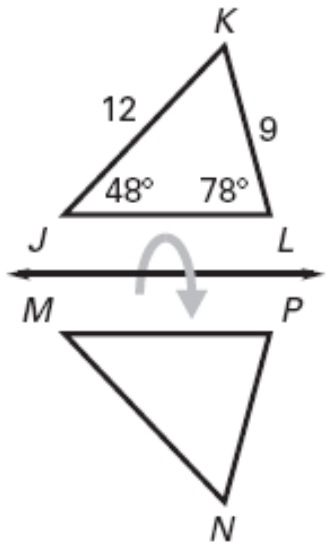
a. No. The image is not congruent to the preimage.

b. Yes. The shaded pentagon is reflected in a line to produce a congruent unshaded pentagon.

c. Yes. The shaded trapezoid is translated down and to the right to form a congruent unshaded trapezoid.

Preserving Length and Angle Measures

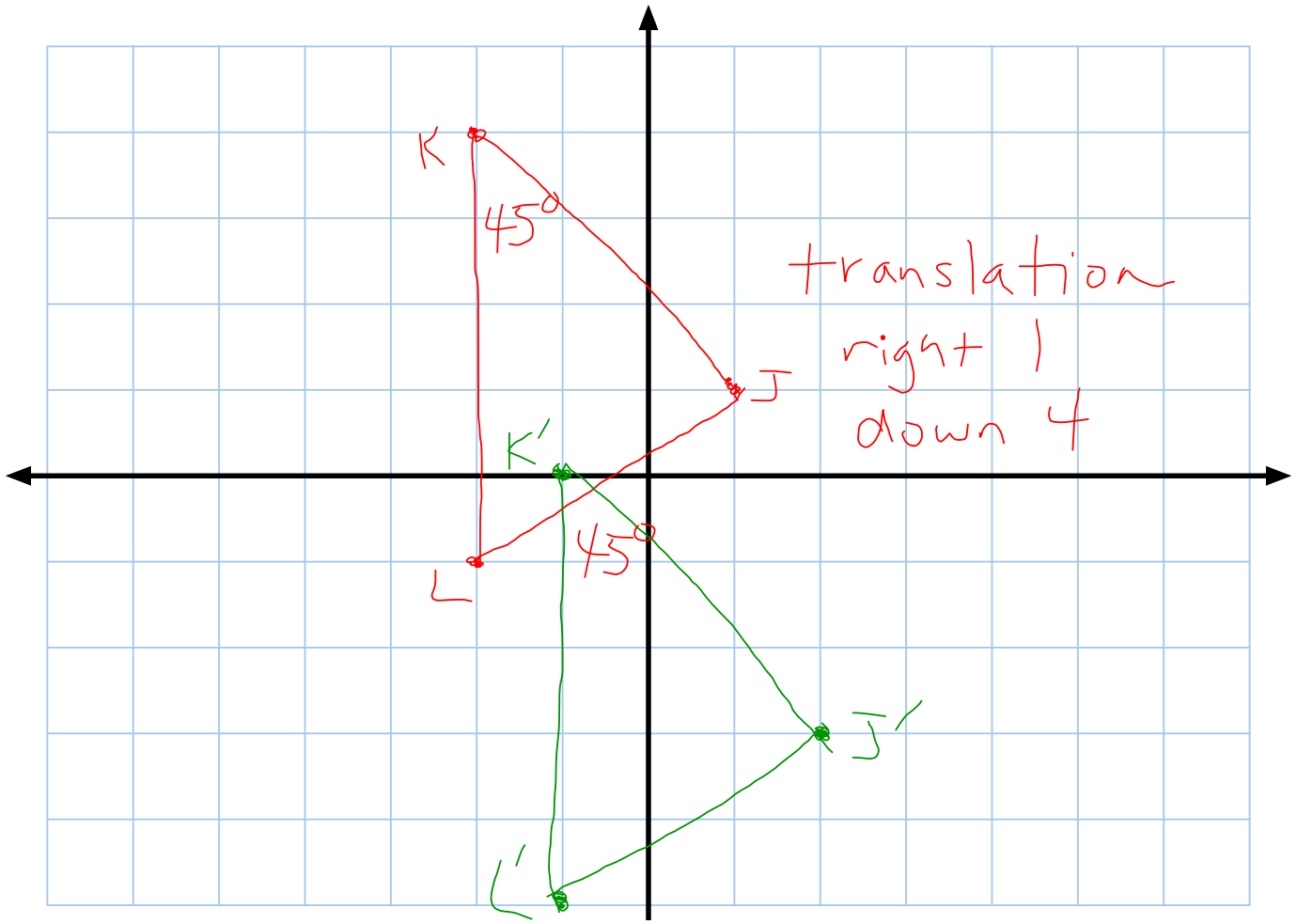
In the diagram, $\triangle JKL$ is mapped onto $\triangle MNP$. The mapping is a reflection. Given that $\triangle JKL \rightarrow \triangle MNP$ is an isometry, find the length of $\overline{NP}$ and the measure of $\angle M$.



Solution

The statement "$\triangle JKL \rightarrow \triangle MNP$" implies that $J \rightarrow$ M, $K \rightarrow$ N, and $L \rightarrow$ P. Because the transformation is an isometry, the two triangles are congruent.

Answer So, $NP =$ KL $=$ 9 and $m\angle M = m\angle$ J $=$ 48°.



The coordinates of $\triangle JKL$ are $J(1, 1)$, $K(-2, 4)$, $L(-2, -1)$. The coordinates of $\triangle J'K'L'$ are $J'(2, -3)$, $K'(-1, 0)$, and $L'(-1, -5)$.

1. Name and describe the transformation.

2. Is this an isometry? yes

3. If $m\angle K = 45°$, find $m\angle K'$.

Assignment

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